TTY Laboratory Of Machine Dynamics
24410 Mechanical Vibrations, 2004

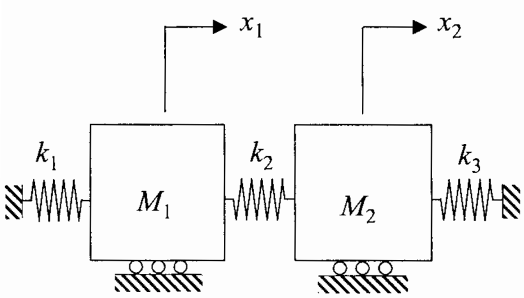
EXAM

January 2005

Instructions: Carry out 3 of the following 4 problems. Access to course notes and a hand calculator is allowed.

Problem 1

Consider the 2-d.o.f. system shown.



$k_1 = 10$ kN/m $\quad M_1 = .1$ kg

$k_2 = 30$ kN/m $\quad M_2 = .1$ kg

$k_3 = 10$ kN/m

(a) Draw free body diagrams of each mass.

(b) Express the equations of motion in matrix form, and identify the mass and stiffness matrices.

(c) Determine the natural frequencies (and express these in Hz) and mode shapes.

(d) Sketch the mode shapes.

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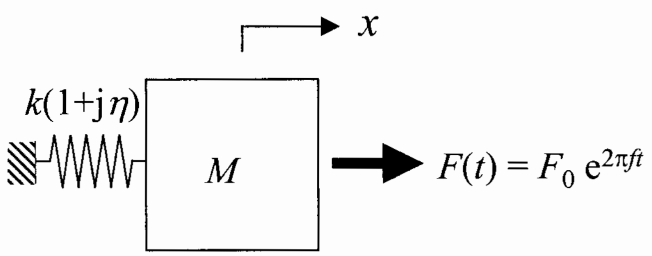
Problem 2

Consider the system illustrated: a mass M restrained by the spring $k(1+j\eta)$ and excited by the force $F(t)$, with parameters as specified in the illustration. The system is thus damped by structural damping.



$$k = 8 \text{ kN/m}, \quad M = 3 \text{ kg}, \quad \eta = .08, \; F_0 = 1 \text{ kN}$$

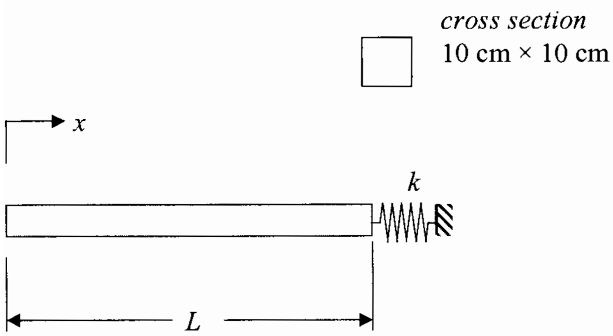
(a) Find the undamped natural frequency f_n, in Hz.

(b) Determine the equivalent viscous damping c_{eq} which is applicable for the cases: $f = 5$ Hz, $f = f_n$, and $f = 10$ Hz.

(c) Find the steady-state response amplitude at the same three frequencies.

Problem 3

Consider the axially-vibrating bar illustrated, free at its left end, and restrained by a spring at its right end.



Suppose that $L = 12$ m, and $k = 200\ 000$ kN/m (i.e., $2 \cdot 10^8$ N/m).

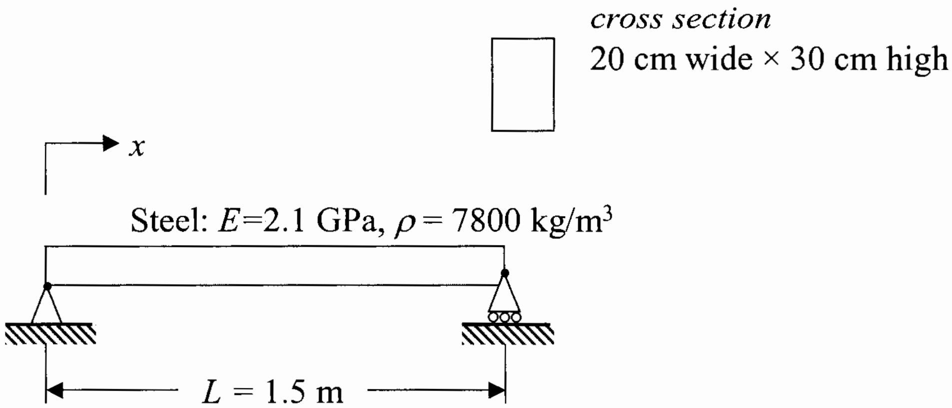
(a) Express the axial boundary conditions at each end.

(b) Derive the frequency equation for axial vibrations. (Note: it is not sufficient to merely state the frequency equation; it must be fully derived on the basis of the boundary conditions)

(c) Find the wave speed (for axial motions).

Problem 4

Consider a pinned-pinned beam, with the dimensions and material properties illustrated.



(a) Write the four boundary conditions.

(b) Derive the frequency equation, for bending vibrations. (note: it is not sufficient to merely state the frequency equation; it must be fully derived on the basis of the boundary conditions)

(c) Find the first 3 natural frequencies.