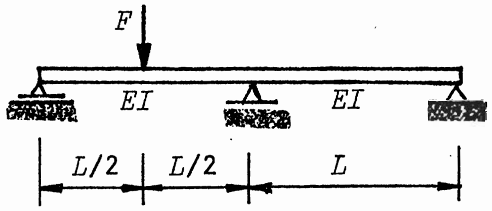
**TTY/** Konstruktiotekniikan laitos
*Teknillinen mekaniikka ja optimointi*

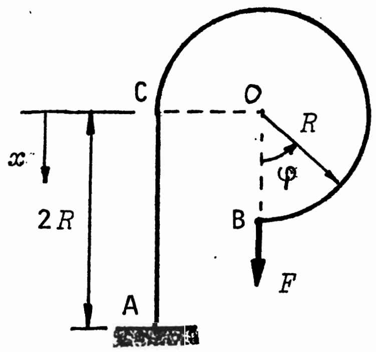
**T1** / TME2200/K2010
*Tapio Salmi*

**TME-2200 Rakenteiden mekaniikka**

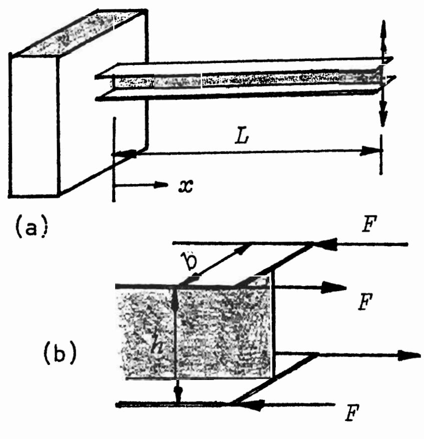
Tentti **1** 24.5.2010



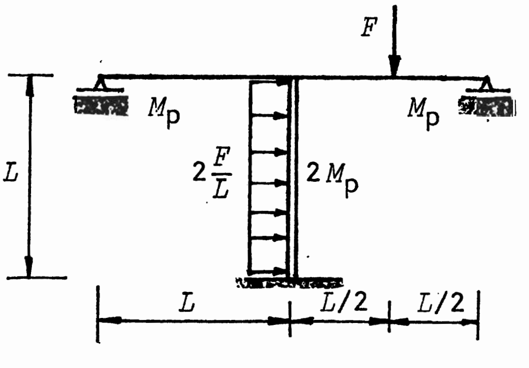

1. Määritä kuvan jatkuvan palkin tukireaktiot ja taipuma voiman $F$ kohdalla integroimalla *kimmoviivan differentiaaliyhtälöä* ja käyttämällä *kärkisulkeisfunktioita*.




2. Kuvan rakenne koostuu ¾-ympyränkaaresta BC ja pystysuorasta pilarista AC. Määritä *CASTIGLIANOn* lauseella pisteen B *vaakasiirtymä*. Vain taivutuksen osuus otetaan huomioon. Rakenneosien taivutusjäykkyys on vakio $EI$.




3. Kuvan ulokepalkin poikkileikkauksen uuman korkeus on $h$ ja laippojen leveys $b$. Määritä palkin bimomentin lauseke $B(x)$ ja vapaan pään kiertymä. Palkin poikkileikkauksen jäykkyydet $GI_v$ ja $EI_\omega$ tunnetaan.




4. Määritä kuvan kehän rajakuormitus *kinemaattisella menetelmällä*. Käytä mekanismia, jossa pystypalkilla on plastisia niveliä, joiden paikat ovat tuloksen tarkkuuden kannalta parhaimmat mahdolliset.

**KÄÄNNÄ!**

5. Valitse oheisista vastausvaihtoehdoista se yksi, jota pidät parhaimpana:

(1) Staattisella menetelmällä saadaan aina liian suuri arvo rajakuormituksen estimaatille.
(2) Neliöpoikkileikkaus ei koe väännössä deplanaatiota.
(3) Avoimen, ohutseinäisen sauvan leikkauskeskiö ja vääntökeskiö yhtyvät.
(4) Komplementtipotentiaalienergian minimilausetta käytettäessä riippumattomina muuttujina ovat siirtymäsuureet.
(5) Kärkisulkeisfunktioita ei voida käyttää, jos palkin poikkileikkaus muuttuu epäjatkuvasti.

Oikeasta vastauksesta saa +2 pistettä, väärästä –1 pisteen ja vastaamattomuudesta nollan.

$$\int \sin^2 x \, \mathrm{d}x = \tfrac{1}{2} x - \tfrac{1}{4} \sin(2x) + C$$

$$\int \cos^2 x \, \mathrm{d}x = \tfrac{1}{2} x + \tfrac{1}{4} \sin(2x) + C$$

$$\int \sin x \, \mathrm{d}x = -\cos x + C \quad , \quad \int \cos x \, \mathrm{d}x = \sin x + C$$

$$\sin^2 x + \cos^2 x = 1 \quad , \quad \sin(2x) = 2 \sin x \cos x$$

## 2305410 Palkkirakenteet

Kaavakokoelma/*Tapio Salmi* sl. 2003

$$\mathrm{f}(x) = < x-a >^{\mathrm{n}} = \begin{cases} (x-a)^{\mathrm{n}} & , \; x \geq a \\ 0 & , \; x < a \end{cases}$$

$$\int_{-\infty}^{x} < x-a >_{-1} \mathrm{d}x = < x-a >^{0}$$

$$\frac{\mathrm{d}}{\mathrm{d}x} < x-a >^{0} = < x-a >_{-1} = \begin{cases} 0 \, , & x < a \\ \infty \, , & x = a \\ 0 \, , & x > a \end{cases}$$

$$EI(x)\, v''(x) = -M_{\mathrm{t}}(x)$$

$$\sigma_{\mathrm{x}} = \frac{M_{\mathrm{t}}}{I_{\mathrm{z}}}\, y$$

$$\frac{\mathrm{d}R}{\mathrm{d}x} = \tau_{\mathrm{sx}}\, t = -\frac{Q_{\mathrm{y}} S_{\mathrm{z}}}{I_{\mathrm{z}}} - \frac{Q_{\mathrm{z}} S_{\mathrm{y}}}{I_{\mathrm{y}}} = \tau_{\mathrm{xs}}\, t = q$$

$$\tau_{\mathrm{zx}} = \tau_{\mathrm{xz}} = \frac{\zeta}{z} \tau_{\mathrm{yx}} \frac{\mathrm{d}z}{\mathrm{d}y}$$

$$A^{\lambda} = \iint_A \mathrm{d}A^{\lambda} = \iint_A \lambda(y,z)\, \mathrm{d}A$$

$$y_0^{\lambda} = S_{\mathrm{z}}^{\lambda} / A^{\lambda} \qquad z_0^{\lambda} = S_{\mathrm{y}}^{\lambda} / A^{\lambda}$$

$$S_{\mathrm{y}}^{\lambda} = \iint_A z\, \mathrm{d}A^{\lambda} = \iint_A z\, \lambda(y,z)\, \mathrm{d}A$$

$$S_{\mathrm{z}}^{\lambda} = \iint_A y\, \mathrm{d}A^{\lambda} = \iint_A y\, \lambda(y,z)\, \mathrm{d}A$$

$$I_{\mathrm{z}}^{\mathrm{E}} = \iint_A E(y,z)\, y^2\, \mathrm{d}A$$

$$I_{\mathrm{y}}^{\mathrm{E}} = \iint_A E(y,z)\, z^2\, \mathrm{d}A$$

$$I_{\mathrm{yz}}^{\mathrm{E}} = \iint_A E(y,z)\, yz\, \mathrm{d}A$$

$$I_{\mathrm{z}} = I_{\zeta} + A\, y_0^2$$

$$\varepsilon_{\mathrm{x}} = \frac{N}{A^{\mathrm{E}}}$$

$$\sigma_{\mathrm{x}}(\mathrm{A}) = \frac{E_{\mathrm{A}} N}{A^{\mathrm{E}}} + \frac{E_{\mathrm{A}} M_{\mathrm{t}\zeta}}{I_{\zeta}^{\mathrm{E}}} \eta_{\mathrm{A}} + \frac{E_{\mathrm{A}} M_{\mathrm{t}\eta}}{I_{\eta}^{\mathrm{E}}} \zeta_{\mathrm{A}}$$

$$\tan\beta = \frac{I_{\zeta}^{\mathrm{E}}}{I_{\eta}^{\mathrm{E}}} \tan\alpha$$

$$\delta W = F\, \delta u$$

$$[L_{\sigma}]\{\sigma\} + \{\underline{f}\} = \{0\} \qquad [G]\{\sigma\} = \{\underline{p}\} \qquad \{u\} = \{\underline{u}\} \qquad W = \int_0^u F(\bar{u})\, \mathrm{d}\bar{u}$$

$$\delta W_{\mathrm{u}} = \iint_{S_{\sigma}} \{\underline{p}\}^{\mathrm{T}} \{\delta u\}\, \mathrm{d}S_{\sigma} + \iiint_V \{\underline{f}\}^{\mathrm{T}} \{\delta u\}\, \mathrm{d}V$$

$$\delta W_{\mathrm{s}} = -\iiint_V \{\sigma\}^{\mathrm{T}} \{\delta \varepsilon\}\, \mathrm{d}V$$

$$\delta W_{\mathrm{u}} + \delta W_{\mathrm{s}} = 0 \qquad \delta W_{\mathrm{u}} = \delta W_{\sigma} \qquad \delta W_{\sigma} = \iiint_V \{\sigma\}^{\mathrm{T}} \{\delta \varepsilon\}\, \mathrm{d}V = -\delta W_{\mathrm{s}}$$

$$\delta U = \iiint_V \{\sigma\}^{\mathrm{T}} \{\delta \varepsilon\}\, \mathrm{d}V \qquad \delta U_0 = \{\sigma\}^{\mathrm{T}} \{\delta \varepsilon\} \qquad \delta W_{\mathrm{u}} = \delta U \qquad \delta W_{\mathrm{u}} = -\delta V$$

$$\{\sigma\} = [E]\{\varepsilon\} \qquad U_0 = \tfrac{1}{2}\{\sigma\}^{\mathrm{T}}\{\varepsilon\} = \tfrac{1}{2}\{\varepsilon\}^{\mathrm{T}}[E]\{\varepsilon\} \qquad \pi = U + V$$

$$\delta U_0 = \{\varepsilon\}^{\mathrm{T}}[E]\{\delta \varepsilon\} \qquad \delta \pi = 0 \qquad \delta^2 \pi \geq 0 \qquad \frac{\partial U}{\partial u_{\mathrm{i}}} = F_{\mathrm{i}}$$

$$\sigma_{\mathrm{ij}} = \frac{\partial U_0}{\partial \varepsilon_{\mathrm{ij}}} \qquad U = \tfrac{1}{2} \int_0^L \left( EA\, (u_{,x})^2 + EI\, (v_{,xx})^2 + GA\, \gamma^2 + GI_{\mathrm{v}}\, \Theta^2 \right) \mathrm{d}x$$

$$p_x = \sigma_x\, l + \tau_{xy}\, m + \tau_{xz}\, n \qquad \sigma_{x,x} + \tau_{xy,y} + \tau_{xz,z} + \underline{f_x} = 0$$

$$p_y = \tau_{xy}\, l + \sigma_y\, m + \tau_{yz}\, n \qquad \tau_{xy,x} + \sigma_{y,y} + \tau_{yz,z} + \underline{f_y} = 0$$

$$p_z = \tau_{xz}\, l + \tau_{yz}\, m + \sigma_z\, n \qquad \tau_{xz,x} + \tau_{yz,y} + \sigma_{z,z} + \underline{f_z} = 0$$

$$\varepsilon_x = u_{,x} \qquad \varepsilon_y = v_{,y} \qquad \varepsilon_z = w_{,z}$$

$$\gamma_{xy} = u_{,y} + v_{,x} \qquad \gamma_{yz} = v_{,z} + w_{,y} \qquad \gamma_{xz} = u_{,z} + w_{,x} \qquad \delta W^* = u\, \delta F$$

$$\delta W^* = \{u\}^{\mathrm{T}} \{\delta F\} \qquad \delta W_{\mathrm{u}}^* = \oint\!\!\oint_S \{u\}^{\mathrm{T}} \{\delta p\}\, \mathrm{d}S + \iiint_V \{u\}^{\mathrm{T}} \{\delta f\}\, \mathrm{d}V \qquad \delta W_{\mathrm{u}}^* + \delta W_{\mathrm{s}}^* = 0$$

$$\delta W_{\mathrm{s}}^* = -\iiint_V \{\varepsilon\}^{\mathrm{T}} \{\delta\sigma\}\, \mathrm{d}V \qquad \delta W_{\varepsilon}^* = \iiint_V \{\varepsilon\}^{\mathrm{T}} \{\delta\sigma\}\, \mathrm{d}V \qquad \delta W_{\mathrm{u}}^* = \delta W_{\varepsilon}^*$$

$$\delta W_{\varepsilon}^* = \sum_{\nu=1}^{n} \int_{L_\nu} (\varepsilon_0\, \delta N + \kappa_y\, \delta M_{\mathrm{ty}} + \kappa_z\, \delta M_{\mathrm{tz}} + \gamma_y\, \delta Q_y + \gamma_z\, \delta Q_z + \Theta\, \delta T)\, \mathrm{d}x$$

$$u_{\mathrm{i}} \cdot 1 = \iiint_V \{\varepsilon\}^{\mathrm{T}} \{\overline{\sigma}\}\, \mathrm{d}V$$

$$u_{\mathrm{i}} \cdot 1 = \sum_{\nu=1}^{n} \int_{L_\nu} (\varepsilon_0\, \overline{N} + \kappa_y\, \overline{M}_{\mathrm{ty}} + \kappa_z\, \overline{M}_{\mathrm{tz}} + \gamma_{xy}\, \overline{Q}_y + \gamma_{xz}\, \overline{Q}_z + \Theta\, \overline{T})\, \mathrm{d}x$$

$$u_{\mathrm{i}} \cdot 1 = \sum_{\nu=1}^{n} \int_{L_\nu} \left(\frac{N\,\overline{N}}{EA} + \frac{M_{\mathrm{ty}}\,\overline{M}_{\mathrm{ty}}}{EI_y} + \frac{M_{\mathrm{tz}}\,\overline{M}_{\mathrm{tz}}}{EI_z} + \zeta_y \frac{Q_y\,\overline{Q}_y}{GA} + \zeta_z \frac{Q_z\,\overline{Q}_z}{GA} + \frac{T\,\overline{T}}{GI_v}\right) \mathrm{d}x$$

$$u_{\mathrm{i}} \cdot 1 = \sum_{\nu=1}^{n} \int_{L_\nu} \frac{N\,\overline{N}}{EA}\, \mathrm{d}x \qquad u_{\mathrm{i}} \cdot 1 = \sum_{\nu=1}^{n} \int_{L_\nu} \frac{M_{\mathrm{tz}}\,\overline{M}_{\mathrm{tz}}}{EI_z}\, \mathrm{d}x \qquad [a]\{X\} + \{u_0\} = \{\underline{u}\}$$

$$u_{0\mathrm{i}} = \sum_{\nu=1}^{n} \int_{L_\nu} \frac{M_{\mathrm{t}0}\,\overline{M}_{\mathrm{ti}}}{EI}\, \mathrm{d}x \qquad a_{\mathrm{ij}} = \sum_{\nu=1}^{n} \int_{L_\nu} \frac{\overline{M}_{\mathrm{ti}}\,\overline{M}_{\mathrm{tj}}}{EI}\, \mathrm{d}x \qquad u = \sum_{\nu=1}^{n} \int_{L_\nu} \frac{M_{\mathrm{t}}\,\overline{M}_{\mathrm{tu}}}{EI}\, \mathrm{d}x$$

$$M_{\mathrm{t}}(x) = M_{\mathrm{t}0}(x) + \sum_{\mathrm{i}=1}^{m} X_{\mathrm{i}}\, \overline{M}_{\mathrm{ti}}(x) \qquad v(x) = v_0(x) + \sum_{\mathrm{i}=1}^{m} X_{\mathrm{i}}\, \overline{v}_{\mathrm{i}}(x)$$

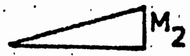
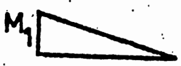
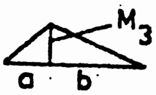
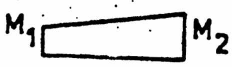
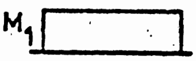
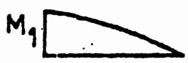
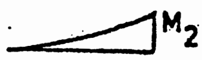
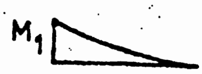
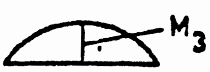
$$\varepsilon_{\mathrm{ij}} = \frac{\partial U_0^*}{\partial \sigma_{\mathrm{ij}}} \qquad \delta U_0^* = \iiint_V \{\varepsilon\}^{\mathrm{T}} \{\delta\sigma\}\, \mathrm{d}V \qquad \delta W_{\mathrm{u}}^* = -\delta V^* \qquad \pi^* = U^* + V^*$$

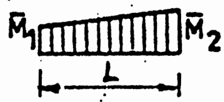
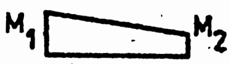
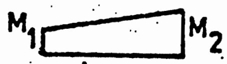
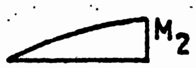
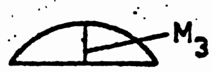
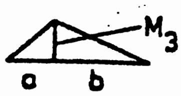
$$\delta \pi^* = 0 \qquad u_{\mathrm{i}} = \frac{\partial U^*}{\partial F_{\mathrm{i}}} \qquad \{F\}^{\mathrm{T}} \{u^P\} = \{P\}^{\mathrm{T}} \{u^F\}$$

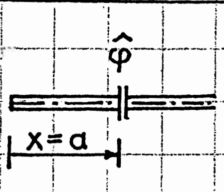
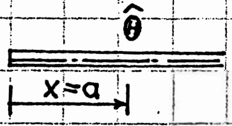
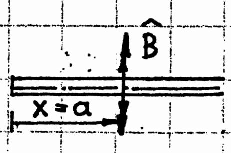
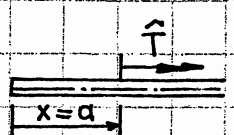
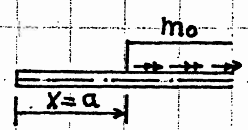
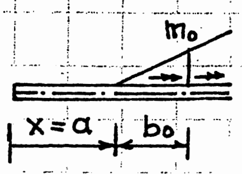
$$U^* = \tfrac{1}{2} \int_L \left(\frac{N^2}{EA} + \frac{M_{\mathrm{ty}}^2}{EI_y} + \frac{M_{\mathrm{tz}}^2}{EI_z} + \zeta_y \frac{Q_y^2}{GA} + \zeta_z \frac{Q_z^2}{GA} + \frac{T^2}{GI_v}\right) \mathrm{d}x$$

Taulukko 1. Mohrin työintegraaleja.

Taulukossa on otsikon viivoitetun kolmio- tai trapetsifunktion $\bar{M}$ ja kullakin rivillä esitetyn kolmio- trapatsi- tai paraabelifunktion M integraali. Kunkin paraabelin huippu on joko integroimisalueen L reunassa tai sen keskellä.

| |   | $\int_0^L \bar{M} M\,dx$ |
|---|---|---|
| 1 |   | $\frac{1}{3} L \bar{M}_2 M_2$ |
| 2 |   | $\frac{1}{6} L \bar{M}_2 M_1$ |
| 3 |   | $\frac{1}{6} L \bar{M}_2 M_3 \left(1 + \frac{a}{L}\right)$ |
| 4 |   | $\frac{1}{6} L \bar{M}_2 (M_1 + 2M_2)$ |
| 5 |   | $\frac{1}{2} L \bar{M}_2 M_1$ |
| 6 |   | $\frac{5}{12} L \bar{M}_2 M_2$ |
| 7 |   | $\frac{1}{4} L \bar{M}_2 M_1$ |
| 8 |   | $\frac{1}{4} L \bar{M}_2 M_2$ |
| 9 |   | $\frac{1}{12} L \bar{M}_2 M_1$ |
| 10 |   | $\frac{1}{3} L \bar{M}_2 M_3$ |

| |   | $\int_0^L \bar{M} M\,dx$ |
|---|---|---|
| 1 |   | $\frac{1}{2} L (\bar{M}_1 + \bar{M}_2) M_1$ |
| 2 |   | $\frac{1}{6} L [\bar{M}_1 (2M_1 + M_2) + \bar{M}_2 (M_1 + 2M_2)]$ |
| 3 |   | Kun $M = \bar{M}$, $\frac{1}{3} L (M_1^2 + M_1 M_2 + M_2^2)$ |
| 4 |   | $\frac{1}{12} L (3\bar{M}_1 + 5\bar{M}_2) M_2$ |
| 5 |   | $\frac{1}{12} L (\bar{M}_1 + 3\bar{M}_2) M_2$ |
| 6 |   | $\frac{1}{3} L (\bar{M}_1 + \bar{M}_2) M_3$ |
| 7 |   | $\frac{1}{6} L \left[\bar{M}_1 \left(1 + \frac{b}{L}\right) + \bar{M}_2 \left(1 + \frac{a}{L}\right)\right] M_3$ |
| | | |
| | | |
| | | |

| Kuormitus<br>Loading | $\hat{\varphi}_i(x)$ ; $\hat{\varphi}(x) = \sum_i \hat{\varphi}_i(x)$ |
|---|---|
|   | $\hat{\varphi} \langle 1 \rangle_a$ |
|   | $\frac{\hat{\theta}}{\lambda} \langle \sinh \lambda (x-a) \rangle_a$ |
|   | $\frac{-\hat{B}}{GI_v} \langle 1 - \cosh \lambda (x-a) \rangle_a$ |
|   | $\frac{-\hat{T}}{GI_v \lambda} \langle \lambda (x-a) - \sinh \lambda (x-a) \rangle_a$ |
|   | $\frac{m_o}{GI_v \lambda^2} \langle \cosh \lambda (x-a) - 1 - \frac{[\lambda (x-a)]^2}{2} \rangle_a$ |
|   | $\frac{m_o / b_o}{GI_v \lambda^3} \langle \sinh \lambda (x-a) - \lambda (x-a) - \frac{[\lambda (x-a)]^3}{6} \rangle_a$ |

$$\Theta = \varphi_{,x} \quad , \quad \Theta = T / GI_v \quad , \quad \nabla^2(\ ) = (\ )_{,xx} + (\ )_{,yy}$$

$$u = \Theta f(y,z) \quad , \quad v = -\Theta xz \quad , \quad w = \Theta xy$$

$$\begin{cases} \nabla^2 f(y,z) = 0 & , \quad (y,z) \in A \\ f_{,n} = z n_y - y n_z & , \quad (y,z) \in \partial A \end{cases}$$

$$I_v = I_p + \iint_A (y f_{,z} - z f_{,y}) dA \quad , \quad I_p = \iint_A r^2 \, dA$$

$$\tau_{xy} = \Phi_{,z} \quad , \quad \tau_{xz} = -\Phi_{,y} \quad , \quad \nabla^2 \Phi = -2 G\Theta$$

$$T = 2 \iint_{\partial A} (-y n_y - z n_z) \Phi \, dA + 2 \iint_A \Phi \, dA \quad , \quad T = 2 \iint_A \Phi \, dA$$

$$T = 2 \iint_A \Phi \, dA + 2 \sum_{i=1}^{n} \Phi_i A_i \quad , \quad T = 2 \iint_{A^*} \Phi \, dA$$

$$\phi = \Phi / G\Theta \quad , \quad I_v = 2 \iint_{A^*} \phi \, dA \quad , \quad \nabla^2 \phi = -2$$

$$W_v = \frac{I_v}{\max |\nabla \phi|} \quad , \quad \tau_x = G\Theta |\nabla \phi|$$

$$\Psi = \iint_A [(f_{,y} - z)^2 + (f_{,z} + y)^2] \, dA \quad , \quad I_v = \Psi \quad , \quad I_v \leq \tilde{\Psi}(\tilde{f}(y,z))$$

$$\Psi^* = \iint_A (\phi_{,y}^2 + \phi_{,z}^2 - 4\phi) \, dA - 4 \sum_{i=1}^{n} \phi_i A_i \quad , \quad I_v = -\Psi^* \quad , \quad I_v \geq -\tilde{\Psi}^*(\tilde{\phi}(y,z))$$

$$-\tilde{\Psi}^* \leq I_v \leq \tilde{\Psi} \quad , \quad \oint_c \vec{\tau}_x \cdot d\vec{s} = G\Theta \cdot 2A$$

$$I_v = 4A^2 / \oint_c \frac{ds}{t} \quad , \quad \tau_x = T / 2At \quad , \quad W_v = 2 A t_{min}$$

$$K_{ki} = \oint \frac{ds}{t_{ki}} \quad , \quad K_{ki} = \frac{s_{ki}}{t_{ki}}$$

$$\sum_{\substack{i=0 \\ i \neq k}} K_{ki} (\phi_k - \phi_i) = 2 A_k \quad , \quad k = 1, \ldots, n$$

$$I_v = 2 \iint \phi \, dA \approx 2 (\phi_1 A_1 + \phi_2 A_2 + \cdots + \phi_n A_n) \quad , \quad W_v = \frac{I_v}{\max \left| \dfrac{\phi_k - \phi_i}{t_{ki}} \right|}$$

$$\omega^1 - \omega^2 = (z_N^1 - z_N^2)(y - y_B) - (y_N^1 - y_N^2)(z - z_B) \quad , \quad A_\omega = A = \iint_A \omega^0 \, dA = \oint t \, ds$$

$$S_\omega = \iint_A \omega \, dA = \oint \omega \, t \, ds \quad , \quad I_\omega = \iint_A \omega^2 \, dA = \oint \omega^2 \, t \, ds$$

$$I_{y\omega} = \iint_A y \, \omega \, dA = \oint y \, \omega \, t \, ds \quad , \quad I_{z\omega} = \iint_A z \, \omega \, dA = \oint z \, \omega \, t \, ds$$

$$z_N = z_N^a - I_{y\omega^a}/I_z \quad , \quad y_N = y_N^a + I_{z\omega^a}/I_y$$

$$\omega_0 - \iint_A \omega^B \, dA / A \quad , \quad \omega = \omega^B + \omega_0$$

$$u = -\Theta \omega \quad , \quad \sigma_\omega = -E\varphi,_{xx} \omega \quad , \quad \tau_\omega = \frac{E\varphi,_{xxx} S_\omega(s)}{t(s)}$$

$$T = T_{SV} + T_\omega \quad , \quad T_{SV} = GI_v \, \Theta = GI_v \, \varphi,_x$$

$$I_v \approx \frac{1}{3} \oint t^3 \, ds = \frac{1}{3} \sum_i t_i^3 s_i \quad , \quad G = E / 2(1+\nu)$$

$$T_\omega = -EI_\omega \varphi,_{xxx} \quad , \quad \lambda^2 = GI_v / EI_\omega$$

$$\varphi,_{xxxx} - \lambda^2 \varphi,_{xx} = m / EI_\omega \quad , \quad T,_x = -m(x)$$

$$\varphi,_{xxx} - \lambda^2 \varphi,_x = -T / EI_\omega \quad , \quad \Theta,_{xx} - \lambda^2 \Theta = -T / EI_\omega$$

$$N = EA u_0,_x \quad , \quad M_{ty} = -EI_y w,_{xx} \quad , \quad M_{tz} = -EI_z v,_{xx}$$

$$B = \iint_A \omega \sigma_x \, dA = -EI_\omega \varphi,_{xx} \quad , \quad \sigma_\omega = \frac{B}{I_\omega} \omega$$

$$\sigma = \frac{N}{A} + \frac{M_{tz}}{I_z} y + \frac{M_{ty}}{I_y} z + \frac{B}{I_\omega} \omega$$

$$\tau = -\frac{Q_y S_y(s)}{I_y \, t(s)} - \frac{Q_z S_z(s)}{I_z \, t(s)} - \frac{T_\omega S_\omega(s)}{I_\omega t(s)}$$

$$\varphi(x) = \varphi_0 + \frac{\Theta_0}{\lambda} \sinh \lambda x - \frac{B_0}{GI_v}(\cosh \lambda x - 1) - \frac{T_0}{GI_v \lambda}(\sinh \lambda x - \lambda x) + \hat{\varphi}(x)$$

$$\langle f(x) \rangle_a = \begin{cases} 0 & , \text{kun } x < a \\ f(x) & , \text{kun } x > a \end{cases}$$

EULER II: $P_n = \pi^2 EI / L^2 \quad , \quad k^2 = P/EI$

$$v,_{xxxx} + k^2 v,_{xx} = k^2 q / P$$