TTY/ Konstruktiotekniikan laitos

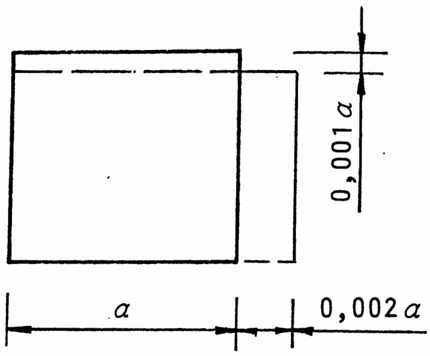
Teknillinen mekaniikka ja optimointi

TME-2100 Materiaalien mekaniikka

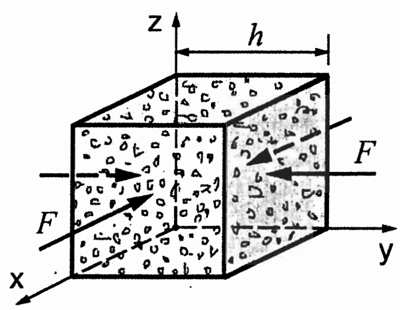
Tentti **2** 18.2.2010

T2/TME-2100/S2009

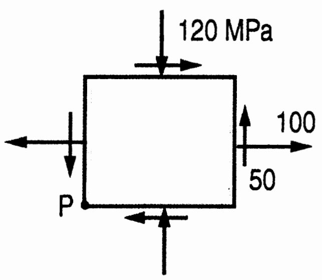
Tapio Salmi



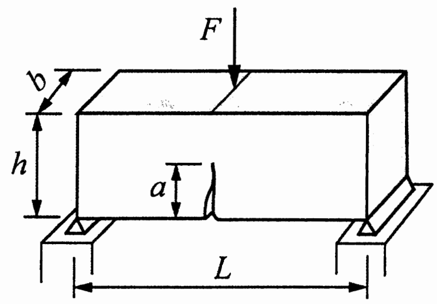
1. Kuvan neliölevy deformoituu tasossaan siten, että sen vaakasuora sivupari pitenee 2 promillea ja pystysuora sivupari lyhenee promillen. Määritä analyyttisesti laskemalla (ei trigonometrisesti) neliön lävistäjien AD ja BC välisen suoran kulman liukuma.



2. Betonikuutiota, jonka särmä $h = 100\ \text{mm}$ ja lujuusluokka K40, puristetaan vaakatasossa olevilla voimilla $F = 90\ \text{kN}$. Laske kuution itseisarvoltaan suurin leikkausjännitys. Paljonko kuution tilavuus muuttuu, kun materiaalikertoimet ovat $E = 32\ \text{GPa}$ ja $\nu = 0{,}10$?



3. Valuraudan vetomurtolujuus on 280 MPa ja puristusmurtolujuus 420 MPa. Määritä materiaalin varmuus murtumisen suhteen, kun rakenneosan eniten rasitetun pisteen jännitystila on kuvan mukainen. Käytä *MOHR-COULOMB*in murtumishypoteesia.



4. Metalliseoksesta tehty kuvan mukainen säröllinen koekappale murtui kuormittavan voiman arvolla $F = 53{,}2\ \text{kN}$. Laske materiaalin murtumissitkeys. $L = 200\ \text{mm}$, $b = 50\ \text{mm}$, $h = 100\ \text{mm}$, $a = 53\ \text{mm}$

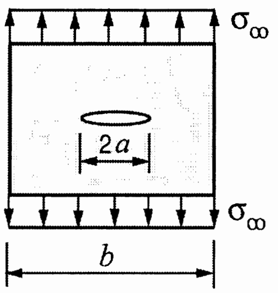
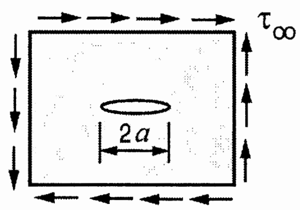
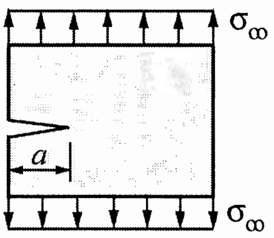
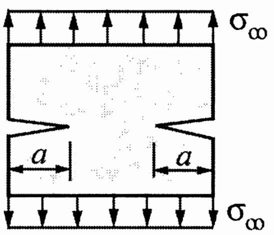
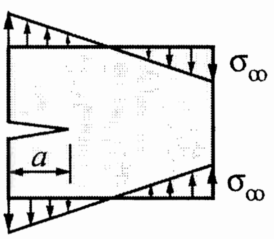
KÄÄNNÄ!

5. Valitse seuraavista vaihtoehdoista se yksi, jota pidät parhaimpana:

(1) Pääjännitysten on aina oltava positiivisia.
(2) Maksimileikkausjännitys esiintyy jännityselementin tahkossa, jossa ei ole normaalijännitystä.
(3) Muodonmuutostilan pääinvariantit riippuvan valitusta koordinaatistosta.
(4) Teräksen murtumissitkeys pienenee teräksen lujuuden kasvaessa.
(5) Paksun levyn läpisärön kärjessä vallitsee tasojännitystila.

Oikeasta vastauksesta saa +3 pistettä ja väärästä -1 pisteen. Vastaamattomuudesta saa nollan.

Taulukko 1 Jännitysintensiteettikertoimia

1		$K_\mathrm{I} = \sigma_\infty \sqrt{\pi a}\ \dfrac{1-(a/b)+1{,}304\,(a/b)^2}{\sqrt{1-2\,a/b}}$ $K_\mathrm{I} \approx \sigma_\infty \sqrt{\pi a}$, kun $a/b \ll 1$
2		$K_\mathrm{II} = \tau_\infty \sqrt{\pi a}$
3		$a/b < 0{,}7$ $K_\mathrm{I} = \sigma_\infty \sqrt{\pi a}\ (1{,}12 - 0{,}23(a/b) + 10{,}6(a/b)^2 +$ $-21{,}7\,(a/b)^3 + 30{,}4\,(a/b)^4)$ $K_\mathrm{I} \approx 1{,}12\,\sigma_\infty \sqrt{\pi a}$, kun $a/b \ll 1$
4		$K_\mathrm{I} = \sigma_\infty \sqrt{\pi a}\ \dfrac{1{,}12 - 1{,}22\,(a/b) + 1{,}04\,(a/b)^3}{\sqrt{1-2\,a/b}}$ $K_\mathrm{I} \approx 1{,}12\,\sigma_\infty \sqrt{\pi a}$, kun $a/b \ll 1$
5		$a/b < 0{,}7$ $K_\mathrm{I} = \sigma_\infty \sqrt{\pi a}\ (1{,}12 - 1{,}39\,(a/b) + 7{,}3\,(a/b)^2 +$ $-13\,(a/b)^3 + 14\,(a/b)^4)$ $K_\mathrm{I} \approx 1{,}12\,\sigma_\infty \sqrt{\pi a}$, kun $a/b \ll 1$

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Kk/TME-2100/S09
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TME-2100 Materiaalien mekaniikka

KAAVAKOKOELMA

$\mathbf{t} = \vec{\sigma} + \vec{\tau}$ $\quad \vec{\sigma} = \mathrm{d}\mathbf{N}/\mathrm{d}A$ $\quad \vec{\tau} = \mathrm{d}\mathbf{Q}/\mathrm{d}A$ $\quad \{t_\alpha\} = [S]\{n_\alpha\}$

$$[S] = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix} \qquad \{n\} = \begin{bmatrix} l \\ m \\ n \end{bmatrix} \qquad l^2 + m^2 + n^2 = 1 \qquad \mathbf{n} = l\,\mathbf{i} + m\,\mathbf{j} + n\,\mathbf{k}$$

$\tau_{ij} = \tau_{ji}$ $\quad \sigma_\alpha = \mathbf{t}_\alpha \bullet \mathbf{n}_\alpha$ $\quad \vec{\tau}_\alpha = \mathbf{t}_\alpha - \vec{\sigma}_\alpha$ $\quad \sigma_\alpha = \{n_\alpha\}^{\mathrm{T}}[S]\{n_\alpha\}$

$t_{\alpha i} = \sigma_{ij} n_j$ $\quad \mathbf{t}_{\bar\alpha} = \sigma_{\bar\alpha}\,\mathbf{n}_{\bar\alpha}$ $\quad \sigma_\alpha = \sigma_{ij} n_i n_j$ $\quad \sigma_{ij} n_j = \sigma_{\bar\alpha} n_i$

$[S]\{n_{\bar\alpha}\} = \sigma_{\bar\alpha}\{n_{\bar\alpha}\}$ $\quad I_1^\sigma = \mathrm{trace}[S] = \sigma_x + \sigma_y + \sigma_z$ $\quad I_3^\sigma = \det[S]$

$$I_2^\sigma = \begin{vmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{vmatrix} + \begin{vmatrix} \sigma_x & \tau_{xz} \\ \tau_{xz} & \sigma_z \end{vmatrix} + \begin{vmatrix} \sigma_y & \tau_{yz} \\ \tau_{yz} & \sigma_z \end{vmatrix} \qquad \sigma_{\bar\alpha}^3 - I_1^\sigma \sigma_{\bar\alpha}^2 + I_2^\sigma \sigma_{\bar\alpha} - I_3^\sigma = 0$$

$I_1^\sigma = \sigma_1 + \sigma_2 + \sigma_3$ $\quad I_2^\sigma = \sigma_1\sigma_2 + \sigma_1\sigma_3 + \sigma_2\sigma_3$ $\quad I_3^\sigma = \sigma_1\sigma_2\sigma_3$

$\sigma_{\hat x} = \sigma_x \cos^2\alpha + \sigma_y \sin^2\alpha + \tau_{xy}\sin(2\alpha)$ $\quad \tau_{\hat x \hat y} = -\frac{1}{2}(\sigma_x - \sigma_y)\sin(2\alpha) + \tau_{xy}\cos(2\alpha)$

$\sigma_k = \frac{1}{2}(\sigma_x + \sigma_y)$ $\quad R = \sqrt{\frac{1}{4}(\sigma_x - \sigma_y)^2 + \tau_{xy}^2}$ $\quad \sigma_{\bar\alpha} = \sigma_k \pm R$

$\tan(2\bar\alpha) = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y}$ $\quad \tau_{xy}\sin(2\bar\alpha) > 0$ $\quad \tau_{\max} = \frac{1}{2}(\sigma_{\mathrm{I}} - \sigma_{\mathrm{III}})$

$\sigma_{\mathrm{okt}} = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) = \frac{1}{3} I_1^\sigma = \frac{1}{3}\sigma_{kk}$ $\quad \tau_{\mathrm{okt}} = \frac{\sqrt{2}}{3}\sqrt{I_1^2 - 3I_3}$

$\tau_{\mathrm{okt}}^2 = \frac{1}{9}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]$ $\quad [S] = [S_{\mathrm{p}}] + [S_{\mathrm{D}}]$

$\sigma_k \equiv p = \frac{1}{3}(\sigma_x + \sigma_y + \sigma_z) = \frac{1}{3} I_1^\sigma$ $\quad [S_{\mathrm{D}}] = (s_{ij})$ $\quad [S_{\mathrm{p}}] = (p\,\delta_{ij})$

$s_{ij} = \sigma_{ij} - p\,\delta_{ij}$ $\quad s_{\bar\alpha} = \sigma_{\bar\alpha} - \sigma_k$ $\quad J_1^\sigma = 0$ $\quad -J_2^\sigma = \frac{1}{2} s_{ij} s_{ij}$

$-J_2^\sigma = \frac{1}{2}(s_1^2 + s_2^2 + s_3^2)$ $\quad -J_2^\sigma = \frac{1}{3}(I_1^2 - 3I_2)$ $\quad \tau_{\mathrm{okt}} = \sqrt{\frac{2}{3}(-J_2^\sigma)}$

$J_3^\sigma = \frac{1}{27}(2I_1^3 - 9I_1 I_2 + 27 I_3)$ $\quad s_{\bar\alpha}^3 + J_2^\sigma s_{\bar\alpha} - J_3^\sigma = 0$ $\quad s_{\bar\alpha} = 2\sqrt{-\frac{1}{3}J_2^\sigma}\cos\theta$

$\cos(2\theta) = \frac{1}{2} J_3^\sigma (-\frac{1}{3} J_2^\sigma)^{-3/2}$ $\quad \sigma_i = s_i + \sigma_k$

$\sigma_{x,x} + \tau_{xy,y} + \tau_{xz,z} + f_x = 0$

$\tau_{xy,x} + \sigma_{y,y} + \tau_{yz,z} + f_y = 0$ $\quad [S]\{n\} = \{\underline{t}\},\ \mathrm{P} \in S_\sigma\ ,\ \{u\} = \{\underline{u}\},\ \mathrm{P} \in S_u$

$\tau_{xz,x} + \tau_{yz,y} + \sigma_{z,z} + f_z = 0$

$\mathbf{u}(\mathrm{P}) = \mathbf{r}(\mathrm{P}) - \mathbf{r}(\hat{\mathrm{P}})$ $\quad \mathbf{u} = u\,\mathbf{i} + v\,\mathbf{j} + w\,\mathbf{k}$ $\quad u = \hat{x} - x\ ,\ v = \hat{y} - y\ ,\ w = \hat{z} - z$

$\hat{x} = \hat{x}(x,y,z)\ ,\ \hat{y} = \hat{y}(x,y,z)\ ,\ \hat{z} = \hat{z}(x,y,z)$ $\quad \mathrm{d}\mathbf{u} = \mathrm{d}u\,\mathbf{i} + \mathrm{d}v\,\mathbf{j} + \mathrm{d}w\,\mathbf{k}$

$\mathbf{u}(\mathbf{r} + \mathrm{d}\mathbf{r}) = \mathbf{u}(\mathbf{r}) + \mathrm{d}\mathbf{u}$ $\quad \mathbf{r} = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$ $\quad \mathrm{d}\mathbf{r} = \mathrm{d}x\,\mathbf{i} + \mathrm{d}y\,\mathbf{j} + \mathrm{d}z\,\mathbf{k}$

$$\{\mathrm{d}u\} = [D]\{\mathrm{d}r\} \qquad [D] = \begin{bmatrix} \partial u/\partial x & \partial u/\partial y & \partial u/\partial z \\ \partial v/\partial x & \partial v/\partial y & \partial v/\partial z \\ \partial w/\partial x & \partial w/\partial y & \partial w/\partial z \end{bmatrix}$$

$(1+\varepsilon)^2 = 1 + 2\{e\}^{\mathrm{T}}[D]\{e\} + ([D]\{e\})^2$ $\quad \tilde{\varepsilon} = \{e\}^{\mathrm{T}}[D]\{e\}$

$[D] = [V] + [\Omega]$ $\quad \tilde{\varepsilon} = \{e\}^{\mathrm{T}}[V]\{e\}$ $\quad \{e\}^{\mathrm{T}}[\Omega]\{e\} = 0$

$$[V] = \begin{bmatrix} \varepsilon_x & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{xy} & \varepsilon_y & \varepsilon_{yz} \\ \varepsilon_{xz} & \varepsilon_{yz} & \varepsilon_z \end{bmatrix} \qquad [\Omega] = \begin{bmatrix} 0 & -\varphi_z & \varphi_y \\ \varphi_z & 0 & -\varphi_x \\ -\varphi_y & \varphi_x & 0 \end{bmatrix}$$

$\varepsilon_x = \partial u/\partial x$ $\quad \varepsilon_y = \partial v/\partial y$ $\quad \varepsilon_z = \partial w/\partial z$ $\quad \gamma_{xy} = \partial u/\partial y + \partial v/\partial x$

$\gamma_{yz} = \partial v/\partial z + \partial w/\partial y$ $\quad \gamma_{xz} = \partial u/\partial z + \partial w/\partial x$

$\gamma_{xy} = 2\,\varepsilon_{xy}$ $\quad \gamma_{yz} = 2\,\varepsilon_{yz}$ $\quad \gamma_{xz} = 2\,\varepsilon_{xz}$ $\quad \sin\phi = \dfrac{|\mathbf{e} \times \mathbf{D}\,\mathbf{e}|}{1+\varepsilon}$

$\phi \approx |\mathbf{e} \times \mathbf{D}\,\mathbf{e}|$ $\quad \varepsilon_{x,yy} + \varepsilon_{y,xx} = 2\,\varepsilon_{xy,xy}$ $\quad \{\mathrm{d}\hat{r}\} = [F]\{\mathrm{d}r\}$

$F_{\mathrm{ij}} = \partial \hat{x}_{\mathrm{i}} / \partial x_{\mathrm{j}}$ $\quad J = \det[F]$ $\quad \mathrm{d}\hat{V} = J\,\mathrm{d}V$ $\quad \mathcal{E} = J - 1$ $\quad \mathcal{E} = \Delta(\mathrm{d}V)/\mathrm{d}V$

$\rho = J\,\hat{\rho}$ $\quad [V]\{\mathrm{e}_{\overline{\alpha}}\} = \varepsilon_{\overline{\alpha}}\{\mathrm{e}_{\overline{\alpha}}\}$ $\quad \mathcal{E} = I_1^{\varepsilon} + I_2^{\varepsilon} + I_3^{\varepsilon} \approx I_1^{\varepsilon}$

$\varepsilon_{\overline{x}} = \varepsilon_x \cos^2\alpha + \varepsilon_y \sin^2\alpha + \varepsilon_{xy} \sin(2\alpha)$ $\quad \{\sigma\} = [E]\{\varepsilon\}$

$\varepsilon_{\overline{x}\overline{y}} = -\tfrac{1}{2}(\varepsilon_x - \varepsilon_y)\sin(2\alpha) + \varepsilon_{xy}\cos(2\alpha)$ $\quad \sigma_{\mathrm{ij}} = \lambda\,\varepsilon_{\mathrm{kk}}\,\delta_{\mathrm{ij}} + 2\,\mu\,\varepsilon_{\mathrm{ij}}$

$$[E] = \frac{E}{(1+\nu)((1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & \overline{\nu} & 0 & 0 \\ 0 & 0 & 0 & 0 & \overline{\nu} & 0 \\ 0 & 0 & 0 & 0 & 0 & \overline{\nu} \end{bmatrix} \qquad \overline{\nu} = \tfrac{1}{2}(1-2\nu)$$

$G = \dfrac{E}{2(1+\nu)}$ $\quad \lambda = \dfrac{\nu E}{(1+\nu)(1-2\nu)}$ $\quad \mu = G = \dfrac{E}{2(1+\nu)}$ $\quad E = \dfrac{\mu(2\mu + 3\lambda)}{\mu + \lambda}$

$\nu = \dfrac{\lambda}{2(\mu+\lambda)}$ $\quad K = \dfrac{E}{3(1-2\nu)}$ $\quad \sigma_{\mathrm{k}} = K\mathcal{E} = 3K\varepsilon_{\mathrm{k}}$ $\quad K = \lambda + \tfrac{2}{3}\mu$

$[S_{\mathrm{p}}] = 3K[V_{\mathrm{p}}]$ $\quad [S_{\mathrm{D}}] = 2G[V_{\mathrm{D}}]$

TME-2100 MATERIAALIEN MEKANIIKKA
Kaavakokoelma 2

$\varepsilon = \varepsilon^{e} + \varepsilon^{p}$ $\qquad d\sigma = E_t\, d\varepsilon$ $\qquad d\sigma = E_p\, \varepsilon^{p}$ $\qquad \dfrac{1}{E_t} = \dfrac{1}{E} + \dfrac{1}{E_p}$

$\kappa \equiv W_p = \int \sigma\, d\varepsilon^{p}$ $\qquad \kappa \equiv \varepsilon_p = \int \sqrt{d\varepsilon^{p}\, d\varepsilon^{p}}$

$\max\{|\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|, |\sigma_1 - \sigma_2|\} = R_e$ $\qquad (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2 R_e^2$

$\tau_{max} = \frac{1}{2}(\sigma_I - \sigma_{III})$ $\qquad \bar{\sigma} = \frac{1}{2}(\sigma_I + \sigma_{III})$ $\qquad R_{-e} = \dfrac{2\bar{c}\cos\varphi}{1 - \sin\varphi}$ $\qquad R_e = \dfrac{2\bar{c}\cos\varphi}{1 + \sin\varphi}$

$m = \dfrac{R_{-e}}{R_e} = \dfrac{1 + \sin\varphi}{1 - \sin\varphi}$ $\qquad m\,\sigma_i - \sigma_j = R_{-e}$, $\sigma_i \ge \sigma_k \ge \sigma_j$, $i, j, k = 1, 2, 3$

$\sqrt{-J_2} - k = 0$ $\qquad \alpha\, I_1 + \sqrt{-J_2} - k = 0$ $\qquad k = \dfrac{2}{\sqrt{3}}\dfrac{R_{-e}}{m+1}$

$(1 - 3\alpha^2)(\sigma_1^2 + \sigma_2^2) - (1 + 6\alpha^2)\sigma_1\sigma_2 + 6k\alpha(\sigma_1 + \sigma_2) = 3k^2$ $\qquad \alpha = \dfrac{1}{\sqrt{3}}\dfrac{m-1}{m+1}$

Suorakulmio: $M_m = \frac{1}{6} b h^2 R_e$ $\qquad M_p = \frac{1}{4} b h^2 R_e$ $\qquad \Phi = M_p / M_m$

$M_p = R_e(\bar{S}_1 + |\bar{S}_2|)$ $\qquad v_{,xx} = -\dfrac{2 R_e / E h}{\sqrt{3}\sqrt{1 - |M|/M_p}}\,\mathrm{sgn}(M)$

$\sigma_x = \dfrac{K_I}{\sqrt{2\pi r}}\cos(\frac{1}{2}\theta)\,(1 - \sin(\frac{1}{2}\theta)\sin(\frac{1}{2}\theta))$ $\qquad \tau_{xy} = \dfrac{K_I}{\sqrt{2\pi r}}\cos(\frac{1}{2}\theta)\sin(\frac{1}{2}\theta)\cos(\frac{1}{2}\theta)$

$\sigma_y = \dfrac{K_I}{\sqrt{2\pi r}}\cos(\frac{1}{2}\theta)\,(1 + \sin(\frac{1}{2}\theta)\sin(\frac{1}{2}\theta))$ $\qquad K_I = \lim_{r \to 0}(\sigma_y(r, 0)\sqrt{2\pi r})$

$\sigma_f = \dfrac{K_I}{\sqrt{\pi a}}$ $\qquad a_t = \dfrac{1}{\pi}\left(\dfrac{K_{Ic}}{R_e}\right)^2$ $\qquad r_p = \dfrac{1}{2\pi}\left(\dfrac{K_I}{R_e}\right)^2$

$\dfrac{da}{dN} = C(\Delta K_I)^n$ $\qquad \dfrac{da}{dN} = C(\eta\,\sigma_a\sqrt{\pi a})^n$

$\sigma = E\,\varepsilon$ $\qquad \sigma = \eta\,\varepsilon^{\bullet}$ $\qquad$ *MAXWELL:* $\varepsilon^{\bullet} = \dfrac{\sigma^{\bullet}}{E} + \dfrac{\sigma}{\eta}$, *KELVIN-VOIGT:* $\varepsilon^{\bullet} + \dfrac{E}{\eta}\varepsilon = \dfrac{\sigma}{\eta}$

Standardimalli: $\varepsilon^{\bullet} + \dfrac{E_2}{\eta}\varepsilon = \dfrac{\sigma}{\eta}(1 + \dfrac{E_2}{E_1}) + \dfrac{\sigma^{\bullet}}{E_1}$ (solidi)