MAT-33500/6 Differentiaaliyhtälöt/Differential equations
Course exam (24 May 2010)

Choose any **four** of the following five problems. Answer in Finnish or English.

1. Consider the general harmonic oscillator (mass= 1)

$$X' = \begin{bmatrix} 0 & 1 \\ -k & -b \end{bmatrix} X,$$

with $b \geq 0$ and $k > 0$. With which values of k, b does the system have i) complex, ii) repeated, or iii) real (and distinct) eigenvalues? Determine the general solution for the case of complex eigenvalues $\alpha \pm i\beta$, and describe the position and velocity of the mass (as function of time) when $b > 0$ and the initial position is $x(0) = 1$ (and initially at rest).

2. Solve the system $X' = AX$, when

$$A = \begin{bmatrix} 0 & 1 & -1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}.$$

3. **a.** Find the general solution of the system

$$X' = AX, \quad A = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}.$$

   b. Show that the origin is an equilibrium point of the nonlinear system

$$x' = -5x + 4y + x^2y, \qquad y' = -2x + y + xy^2,$$

and examine its stability.

4. Consider the nonlinear system

$$x' = xy, \qquad y' = 1 - y^2.$$

Find the equilibrium points, examine their stabilities, and sketch the phase portrait (with some estimated solution curves).

5. Consider the nonlinear system

$$x' = x^2 - y, \qquad y' = 1 - x^2 - y^2.$$

Find the equilibrium points, examine their stabilities, and sketch the phase portrait (with some estimated solution curves).