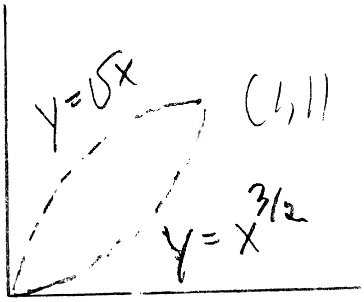
MAT-20400 Vektorianalyysi
Tentti 23.10.2009
Ei laskimia, taulukot jaetaan

1. (i) Laske $\int_C \mathbf{F} \bullet d\mathbf{r}$, kun $\mathbf{F} = [3x^2 + y, \ -yz, \ 2xyz]$ ja C on $\mathbf{r}(t) = (t, \sqrt{t}, 1/\sqrt{t})$ missä $t \in [1,2]$.

(ii) Olkoon $\mathbf{F}(x,y) = \left(xy + x + y^2, x^2 - x + 1\right)$ ja käyrä C kuten kuvassa.
Laske $\oint_C \mathbf{F} \bullet d\mathbf{r}$.



2. (i) Määrää $\mathbf{F} = (yz+1)\mathbf{i} + xz\mathbf{j} + xy\mathbf{k}$ skalaaripotentiaali (mikäli on olemassa).

(ii) Laske $\int_C \mathbf{F} \bullet d\mathbf{r}$, kun C on jana pisteestä (1, 1, 1) pisteeseen $(\sqrt{2}, \sqrt{3}, \sqrt{6})$. (**F** (i)-kohdasta)

3. Laske vektorikentän $\mathbf{F} = [2xy, x^2 - y^2, -3z]$ vuo puolipallon

$$R = \left\{(x,y,z) : x^2 + y^2 + (z-2)^2 = 4, z \geq 2\right\}$$

pinnan läpi. (Huom! Vain puolipallon pinta).

4. Olkoon $\mathbf{F}(x, y, z) = y\mathbf{i} + 5xz\mathbf{j} - 3xyz\mathbf{k}$.

Laske $\iint_S (\nabla \times \mathbf{F}) \bullet \mathbf{n} \, d\sigma$, kun **n** on S:n yksikkönormaali, jonka **k** koordinaatti on positiivinen (pinnan yläpuoli on positiivinen puoli) ja S on puolipallon pinta:

$x^2 + y^2 + (z + 3)^2 = 4 \qquad z \geq -3$.

MAT-20400 Vektorianalyysi, kokeen kaavaliite

1. (1) $\nabla \times \nabla f = \mathbf{0}$
   (2) $\nabla \bullet (\nabla \times \mathbf{F}) = 0$
   (3) $\nabla(\mathbf{F} \bullet \mathbf{G}) = \mathbf{F} \times (\nabla \times \mathbf{G}) + \mathbf{G} \times (\nabla \times \mathbf{F}) + (\mathbf{F} \bullet \nabla)\mathbf{G} + (\mathbf{G} \bullet \nabla)\mathbf{F}$
   (4) $\nabla \bullet (\mathbf{F} \times \mathbf{G}) = \mathbf{G} \bullet (\nabla \times \mathbf{F}) - \mathbf{F} \bullet (\nabla \times \mathbf{G})$
   (5) $\nabla \times (\mathbf{F} \times \mathbf{G}) = \mathbf{F}(\nabla \bullet \mathbf{G}) - (\mathbf{F} \bullet \nabla)\mathbf{G} + (\mathbf{G} \bullet \nabla)\mathbf{F} - \mathbf{G}(\nabla \bullet \mathbf{F})$
   (6) $\nabla(fg) = (\nabla f)g + f\nabla g$
   (7) $\nabla \bullet (f\mathbf{G}) = (\nabla f) \bullet \mathbf{G} + f(\nabla \bullet \mathbf{G})$
   (8) $\nabla \times (f\mathbf{G}) = (\nabla f) \times \mathbf{G} + f(\nabla \times \mathbf{G})$
   (9) $\nabla[h(f(\mathbf{r}))] = h'(f(\mathbf{r}))\nabla f(\mathbf{r})$

2. $$\int_C f\,ds = \int_a^b f(\mathbf{r}(t))\|\mathbf{r}'(t)\|\,dt$$

3. $$\int_C \mathbf{F}\bullet d\mathbf{r} = \int_a^b \mathbf{F}(\mathbf{r}(t))\bullet\mathbf{r}'(t)\,dt = \int_C \mathbf{F}\bullet\mathbf{T}\,ds = \int_C f_1\,dx + \int_C f_2\,dy + \int_C f_3\,dz$$

4. $$\oint_{\partial R} \mathbf{F} \bullet d\mathbf{r} = \iint_R \left(\frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y}\right) dx\,dy$$

5. $$\oint_{\partial R} \mathbf{F} \bullet \mathbf{n}\,ds = \iint_R \left(\frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y}\right) dx\,dy$$

6. $$\iint_S f\,dS = \iint_R f(\mathbf{r}(u,v))\|\mathbf{r}_u(u,v) \times \mathbf{r}_v(u,v)\|\,du\,dv$$

7. $$\iint_S f\,dS = \iint_R f(x,y,z(x,y))\sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\,dx\,dy$$

8. $$\iint_S \mathbf{F} \bullet \mathbf{n}\,dS = \iint_R \mathbf{F}(\mathbf{r}(u,v)) \bullet [\mathbf{r}_u(u,v) \times \mathbf{r}_v(u,v)]\,du\,dv$$

9. $$\iint_S \mathbf{F} \bullet \mathbf{n}\,dS = \iint_R \mathbf{F}(x,y,z(x,y)) \bullet \left(-\frac{\partial z}{\partial x}, -\frac{\partial z}{\partial y}, 1\right) dx\,dy$$

10. $$\iint_S \mathbf{F} \bullet \mathbf{n}\,dS = \iiint_T \nabla \bullet \mathbf{F}\,dV$$

11. $$\oint_C \mathbf{F} \bullet d\mathbf{r} = \iint_S (\nabla \times \mathbf{F}) \bullet \mathbf{n}\,dS$$

12. $$f(\mathbf{r}) = \int_{A_0}^{\mathbf{r}} \mathbf{F} \bullet d\mathbf{r}$$

13. $$\begin{cases} x = \rho \sin\phi \cos\theta \\ y = \rho \sin\phi \sin\theta \\ z = \rho\cos\phi \end{cases} \implies dV = \rho^2 \sin\phi\,d\rho\,d\phi\,d\theta$$

$$\sin(2t) = 2\sin t\cos t \qquad \sin^2 t = \frac{1-\cos(2t)}{2} \qquad \cos^2 t = \frac{1+\cos(2t)}{2}$$